The Shapley Value of Games for Power Allocation Used in Cognitive Radio Networks

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Abstract—In this paper, we propose a new method of power allocation (PA) for cognitive radio networks. This method is based on the Shapley value of games. In this work we also take into account the average transmission power constraints for secondary users (SUs). An optimization problem associated with the power control policy is formulated on the basis of a given method. An algorithm is developed for the sake of finding the optimal solution of this problem. The simulation results show the effectiveness of the proposed power control policy.

Index Terms—Cognitive radio networks, game theory, Shapley vector

I. INTRODUCTION

Cognitive radio (CR) networks are highly agile wireless platforms capable of autonomously choosing device parameters based on current conditions [1], [2]. For instance, during any natural disaster in wireless communication, centralized wireless network may not be available due to the overloaded and/or damaged access points or base stations. In the CR networks wireless access can be established in areas of the accessible wireless infrastructure.

Cognitive radio networks provide the capacity to share a wireless channel with the licensed users in an opportunistic way. The CR networks are envisioned to be able to provide a high bandwidth to mobile users via heterogeneous wireless architectures and a dynamic spectrum access (DSA) technology. The users of the CR networks can either communicate with each other in a multi-hop manner or access the base station. Therefore, we may categorise the CR application of the spectrum into three possible scenarios, (a) a CR network on a licensed band, (b) a CR network on an unlicensed band, (c) a CR network on both a licensed band and an unlicensed band.

In this paper, we examine the problem of power allocation (PA) in the CR networks on a licensed and unlicensed band with the main emphasis on the concept of hierarchy of the existing between radios. This problem arises in the following situations: (a) when the primary and secondary systems share the spectrum, (b) when users have access to the medium in an asynchronous manner, (c) when operators deploy their networks at different times, (d) when some nodes have more power than others, such as the base station. One of the most popular model of the hierarchical spectrum of sharing is the Stackelberg equilibrium (SE) [3], [4]. This approach was motivated by the fact that the noncooperative Nash equilibrium (NE) is generally inefficient and nonoptimal. The Stackelberg equilibrium provides better outcomes as compared to the non-cooperative approach. However, the mathematical framework of the Stackelberg equilibrium is not suitable for practical use. Therefore, we propose a new scheme of the power allocation (PA) problem in the CR networks which is based on Shapley’s value vector.

The main goal of this paper is to show that the Shapley values as one of the game’s solutions of power allocation problem in the CR networks. Following the Shapley value model, we will discuss the issues of maximizing the effective throughput of the secondary users (SUs) in a licensed band subject to the constraints of the transmission power and average interference power.

The rest of this paper is outlined as follows. Section 2 concerns the application of the Shapley value of games to power allocation for the SUs in the CR networks. In section 3, we introduce the optimal power policy to maximize the effective capacity of the SUs subject to the constraints of the transmission power and the average interference power. The simulation results are illustrated in section 4. Concluding remarks are drawn in section 5.

II. APPLICATION OF THE SHAPLEY VALUE OF GAMES TO POWER ALLOCATION IN COGNITIVE RADIO NETWORKS

In this section we investigate applications of the Shapley value of games to power allocation in the CR network.

Our approach is based on two important multiuser channel models, namely: the multiple access channel (MAC) [5] and the interference channel (IPC).

In the first one, we assume use of the uplink channel in a single-cell multi-carrier cellular system in which each multiple access channel (MAC) consists of K transmitters aiming to communicate with a single receiver using a common channel. There exist N independent or parallel MACs. None of the transmitters in different MACs interferes with each other. The channel gain from transmitter i to the receiver over channel n is denoted by \( h_{ni} \). Let the channel realizations during the transmission of M consecutive symbols be constant. All the channel realizations, \( i, i \in \{1,\ldots,k\} \) and \( n \in \{1,\ldots,N\} \) are drawn from a Gaussian distribution with a zero mean and a unit variance. Thus, the power allocated by transmitter i to channel n is denoted by \( p_i^n \). We can formulate the following
The condition for the transmitter $i$, namely
\[ \sum_{n=1}^{N} p_{i}^{n} \leq P_{i}^{\text{max}} \quad \forall i \in \{1, \ldots, K\} \quad (1) \]

We assume that the noise at the receiver is described by $w_{i}^{n}$. It corresponds to the additive white Gaussian noise (AWGN) process with a zero mean and variance $\sigma^{2}$. The received signal can be written as
\[ y_{i}^{n} = \sum_{i=1}^{K} h_{i,j}^{n} x_{i,j}^{n} + w_{i}^{n} \quad \forall n \in \{1, \ldots, N\} \quad (2) \]

where $x_{i,j}^{n}$, $h_{i,j}^{n}$ are the transmitted symbols and the channel realization from transmitter $i$ to receiver $j$ on channel $n$, respectively.

Assuming a single-user decoding (SUD) on channel $n$ for transmitter $i$, the received signal for interference plus the noise ratio (SINR) is expressed as
\[ \text{SINR}_{i}^{n} = \frac{p_{j}^{n} | h_{i,j}^{n} |^{2}}{\sum_{j \neq i}^{K} p_{j}^{n} | h_{j,j}^{n} |^{2} + \sigma^{2}} \quad \forall i \in \{1, \ldots, K\} \]

The interference channel model (IFC) described by T. S. Chan et al. [6] and by Etkin et al. [7] consists of a set of $K$ point-to-point links sufficient to produce mutual interference due to their co-existence on the same channel. Assuming that $N \geq 1$ channels are available in the IFC model, independent or parallel channels exist, where transmitters in different IFCs do not interfere with each other. In essence, the IFC model corresponds to the transmission in pairs between nodes over a set of sub-carriers. We assume that the link gain between transmitter $j$ and receiver $k$ is denoted by $h_{j,k}^{n}$, where $n = \{1, \ldots, N\}$, and $(i,j) \in \{1, \ldots, K\}$. Thus, the received signal at receiver $i$ is given by
\[ x_{i}^{n} = \sum_{j=1}^{K} h_{i,j}^{n} x_{i,j}^{n} + w_{i}^{n} \quad (4) \]

where $w_{i} = \{w_{i,1}, \ldots, w_{i,n}\}$ is the noise at receiver $i$ over channel $n$.

Assuming a single-user decoding (SUD) on channel $n$ for transmitter $i$, the received SINR can be expressed as
\[ \text{SINR}_{i}^{n} = \frac{p_{i}^{n} | h_{i,i}^{n} |^{2}}{\sum_{j \neq i}^{K} p_{i}^{n} | h_{j,j}^{n} |^{2} + \sigma^{2}} \quad \forall i \in \{1, \ldots, N\} \quad (5) \]

The difference between Eqs. (3) and (5) is that each transmitter knows the channel realization $h_{i,j}^{n}$ for all $\forall i \in \{1, \ldots, K\}$ in the MAC model and $h_{i,j}^{n}$ for all $(i,j) \in \{1, \ldots, K\}^{2}$ in the IFC model.

In the PA game, the set of players includes transmitters, base stations, and mobile stations. In general, a game is presented in a normal form as follows:

**Definition 1 (Normal form)** [3]
A game in a normal form is given by $\{K, S, \{u_{k}\}_{k \in K}\}$ and is composed of three elements:
- a set of players: $K = \{1, \ldots, K\}$,
- a set of strategy profiles: $S = S_{1} \times \ldots \times S_{k}$, where $S_{k}$ is the strategy set of player $k$,
- a set of utility functions: the $k$-th player’s utility function is $u_{k} : S_{k} \rightarrow R_{+}$ and is denoted by $u_{k}(s_{k}, s_{-k})$ where $s_{k} \in S_{k}$ and $s_{-k} = (s_{1}, \ldots, s_{k-1}, s_{k+1}, \ldots, s_{K}) \in S_{1} \times \ldots \times S_{k-1} \times S_{k+1} \times \ldots \times S_{K}$.

Considering that players are willing to cooperate to achieve a fair allocation of resources, we impose a condition that the utility function must account for both the interference perceived by the current players, and the interference that particular player is causing to neighboring players sharing the same channel.

The utility function is defined as follows:

**Definition 2 (Utility function)**
The utility function may be
\[ u_{k}(s_{k}, s_{-k}) = - \sum_{j \neq k, j=1}^{N} p_{j}(s_{j}) G_{kj} f(s_{j}, s_{k}) \]

where $G_{kj}$ is the link gain between transmitter $j$ and receiver $k$, $f(s_{j}, s_{k})$ is an interference function given by
\[ f(s_{j}, s_{k}) = \begin{cases} 1 & \text{if } s_{j} = s_{k}, \text{ transmitter } j \text{ and } k \text{ choose the same strategy (same channel)} \\ 0 & \text{otherwise} \end{cases} \quad (7) \]

The above utility function accounts for both the interference measured at the current user’s receiver and the interference created by the user to others.

As a solution of the coalition for the game a method introduced by L. S. Shapley is used [8], [9]. The main idea of Shapley’s method lies in the definition of player usefulness for the coalition and rewards assignment which is proportional to their potential contributions.

We introduce the Shapley value

**Definition 3 (The Shapley value of a game in a normal form)**
Let $v$ be a game given by $\{K, S, \{u_{k}\}_{k \in K}\}$. The Shapley value of $v$, $\Phi(v) = (\phi_{1}(v), \ldots, \phi_{K}(v)) \in K$ is defined by
\[ \phi_{i}(v) = (K - 1)! \sum_{|C| = K-1} \frac{(a-1)!}{K!} \mu(C \setminus \{i\}) \quad (8) \]

for each player $i$, $1 \leq i \leq K$ attached to coalition $C$ counting $(a - 1)$ players as the $a$ player, $\sigma(C, i)$ is the usefulness of player $i$ for the coalition $C$ and is given by
\[ \sigma(C, i) = \mu(C) - \mu(C \setminus \{i\}) \quad (9) \]

where $\mu(C \setminus \{i\})$ is the reward for coalition $C$ without the $i$-th player and $\mu(C)$ is the reward for coalition $C$. 
Each coalition can be assigned the usefulness function of all players for the formed coalition. We assume that the usefulness function of a dummy player is equal to 0.

A formal definition of the influence of the outgoing player for the coalition is given as follows:

**Definition 4 (Influence of the player’s going out into the coalition)**

For the sake of the best possible coalition we have observed that the reward of coalition \( C \) changes its value from 1 to 0 after player \( i \) is going out of the coalition.

According to the Shapley method [8], [9] we can univocally assign to each game the imputation which is reasonable partitioning of winnings. The following definition gives the terms of the player’s participation in the coalition.

**Definition 5 (Participation in the coalition)**

The participation of the player in the coalition is determined by the values of the Shapley vector.

### III. The Spectrum Sharing for Cooperative Secondary Systems with the Use of the Shapley Value

In this section, we investigate the dynamic spectrum sharing in the CR network in which primary systems lease the spectrum to secondary system in exchange for cooperation in the PA game.

We assume that a primary transmitter wishes to send information to its primary receiver either directly with a rate \( R_{dir} \) or by means of the cooperation from a subset \( S \subseteq S_{tot} \) of \( |S| = k \leq |S_{tot}| = k \) secondary nodes/transmitters. The primary system can divide its data into two parts \((1 - \alpha)/\text{bit durations, and } \alpha L \text{ bit durations with } 0 \leq \alpha \leq 1\). The first part is dedicated to a direction transmission from primary transmitter to the primary receiver whereas the second \( \alpha L \) bit duration is again divided into two parts. One part, consisting of \( \beta \alpha L \), with \( 0 \leq \beta \leq 1 \), is dedicated to sending information from the primary transmitter to the primary receiver using the secondary nodes by means of the distributed space time coding [10]. The remaining \( \alpha (1 - \beta) L \) bits are devoted to the secondary network for the sake of its own data transmission. The problem of power allocation in the secondary system of the CD network can be solved by maximization of its utility function while deciding about the portion of time-slots \( \alpha, \beta \) and \( S \subseteq S_{tot} \) subset of secondary transmitters.

Given the set \( S \) and cooperation parameters \( \alpha, \beta \), the PA optimization problem is given by

\[
\max_{\alpha, \beta, S} \left( \sum_{k \in S, k=1}^{C} u_k(s_k, s_{-k}) \right)
\]

subject to \( S \subseteq S_{tot}, 0 \leq \alpha, \beta \leq 1 \).

**procedure** power_allocation; compute_the_Shapley_value_for_coalition;
repeat
for \( i := 1 \) to \( C \) do
compute \( p_i \) from Eq.(12);
find \( \hat{p}_i = f_i \cdot \Phi_i \cdot p_i \);
endfor;
until coalition_is_empty;

**Fig. 1.** An algorithm for power allocation in CR network

The secondary system maximizes its utility function of the formed coalition \( C \) by means of maximization of the achievable transmission rate along with taking into consideration the cost of the transmitted energy \( E_c \). The optimization problem for the secondary system can be expressed as

\[
\max_{s_k} \left( \sum_{k=1, k \in S}^{C} u_k(s_k, s_{-k}) \right) = \max_{s_k} \left( \alpha(1 - \beta) \right)
\]

\[
\log_2 \left( 1 + \frac{\sigma^2}{E_c} \left| \sum_{j=1, j \neq i}^{S_{tot}} \left| h_{S,ijk} \right|^2 s_j \right| - E_c s_k \right)
\]

subject to \( 0 \leq s_k \leq S_{i, max} \).

Solving Eq. (11) we can present the value of power for transmitter \( i \), namely

\[
p_i = \max \left( 0, 1 - \beta \frac{\sigma^2}{E_c} \left| \frac{|h_{S,i}|^2}{|h_{S,ii}|^2} \right| - \sum_{j=1, j \neq i}^{S_{tot}} \left| h_{S,ijk} \right|^2 p_j \right)
\]

In our approach the interaction between the primary and secondary users is modelled as cooperative game. The coalition maximizes its own utility function. Using the Shapley value we obtain the participation of each player in the game with the maximal utility function of the coalition.

\[
\hat{p}_i = \arg \max_a (u_i(p_1, \ldots, p_C)) = f_i \cdot \Phi_i (v) \cdot p_i
\]

\[
0 \leq p_i \leq P_{i, max}
\]

where \( \Phi_i(v) \) is the Shapley value for transmitter \( i \) and \( f_i \) is a normalizing parameter. We present an algorithm that first finds a coalition among the transmitters, and further, by means of using the Shapley value calculates the participation of each of them (see Fig. 1). Additionally, our algorithm maximizes the throughput and minimizes the average interference power.

### IV. Simulation Results

In this section, we present the simulation results in order to study the performance of our scheme compared with the Stackelberg equilibria in the same scenario.

We have the following general settings for the simulation. We place the primary transmitter BS at coordinates and five secondary users which are uniformly located in the area \( 100 \, \text{m} \times 100 \, \text{m} \). The maximum power for a secondary user is \( P_{max} = 100 \). The rest of the parameters are set as follows: maximal power 100 mW, threshold power \( 3 \times 10^{-7} \) mW. The...
AWGN at all receivers has the same power $W = 5 \times 10^{-7}$ W and the interference power threshold at all receivers is -50 dB. By means of using the Nash and the Stackelberg equilibria [11] we find the optimal value of $p_1^i$ while keeping $p_2^1, \ldots, p_N^1$ fixed and then we find the optimal $p_1^i$ keeping the other $p_n^i$ $(n \neq 2)$ fixed and so on. Such a process guarantees convergence because each iteration increases the objective function.

We then evaluate how the Shapley vector of the formed coalition affects the power allocation in the CR networks with a varied number of secondary users. Fig. 2 depicts the achievable rate for both types of users versus the signal-to-noise ratio for the SE, NE and Shapley value (SV) approaches. As can be seen, the average achievable rate of Shapley’s vector is comparable to the SE approach.

Figure 3 shows the cumulative distribution function (CDF) of the ratio of the cooperative and noncooperative approach. In this scenario, we assume that $K = 3$ operators with one primary operator and three secondary users sharing the same spectrum. It is composed of $N = 5$ carriers. The entrance of each player to the coalition took place according to their index value. In order to achieve the CDF in a noncooperative approach we propose a repeated game in which the players will be added to the coalition in a strictly defined succession. This succession must guarantee the highest values of Shapley’s vector.

V. CONCLUSION

In this paper, we proposed a new approach to power allocation in the cognitive radio networks. Our approach was based on the Shapley value of games. We also proposed a use of the algorithm which allows us to the maximization of the throughput and minimization of the average interference. From our simulation experiments, we concluded that our model leads to accurate results when the secondary users can form a game. The future work could also consider more extensive simulation of our method.

REFERENCES