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ARE CARRIER TRANSPORT EFFECTS IMPORTANT FOR CHIRP MODELLING OF QUANTUM-WELL LASERS?

In the paper the impact of carrier transport effects on the chirp modelling of quantum-well lasers is investigated. Particularly, the difference between the full modelling based on quantum-well laser rate equations is compared with modelling based on formulas derived for bulk lasers. As it was shown the relations between chirp and intensity modulation are quite similar in both cases.

Keywords: laser chirp, laser modelling

1. INTRODUCTION

The quantum-, or multi-quantum-well (QW, or MQW) structure introduced to the semiconductor laser design implies some new phenomena in the device operation, when compared with the bulk laser design. Among them the transport of injected carriers across the separate-confinement-heterostructure (SCH) and capturing them into the QW region introduce some delay in the carriers flow. Consequently noticeable variations of the concentration of carriers accumulated in SCH region occur. Because a large fraction of the optical mode lies in the SCH, this carrier density variation affects the lasing frequency i.e. introduces a new chirp component.

There are plenty of papers, in which the significant differences in chirp characteristics of bulk and QW lasers are pointed out [1-4]. On the other hand, there are some papers in which the QW laser chirp is modelled using equations derived for bulk device. In some of them the considerations are verified by experiments, that seems to proof such chirp treatment [5-7]. The aim of the work presented herein is to clarify this confusing inconsistency and to pointed out the area, where the simple chirp model may be used for QW lasers.

2. THEORETICAL BASICS

The basic mathematical model of semiconductor laser is the set of rate equations, which describes the dynamics of carrier and photon densities, and relate them to the laser frequency chirp and output optical power.

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2.1. Bulk laser modelling

For the bulk laser the rate equations may be written in following form:

\[
\frac{dN}{dt} = \frac{I}{eV_a} - \frac{N}{\tau_e} - \frac{g_0(N - N_T)}{1 + \varepsilon_g S},
\]

(1)

\[
\frac{dS}{dt} = \frac{\Gamma g_0(N - N_T)}{1 + \varepsilon_g S} S - \frac{S}{\tau_p} + \frac{\Gamma \beta N}{\tau_e},
\]

(2)

\[
\Delta \nu = \frac{\alpha}{4\pi} \Gamma g_0(N - N_{TH}),
\]

(3)

\[
P = \frac{\eta V_a h V_0}{\Gamma \tau_p} S,
\]

(4)

where \( N \) is the carrier concentration in the active region, \( S \) is photon concentration, \( I \) is injected current, \( e \) is the electron charge, \( V_a \) is the active region volume, \( \tau_e \) is carrier lifetime, \( g_0 \) is the differential gain, \( \varepsilon_g \) is the gain compression factor, \( N_T \) is carrier concentration for transparency, \( N_{TH} \) is threshold carrier concentration, \( \Gamma \) is confinement factor, \( \tau_p \) is photon lifetime, \( \beta \) is spontaneous emission coefficient, \( \Delta \nu \) is optical frequency deviation (i.e. the chirp), \( \alpha \) is the line enhancement factor, \( P \) is the output power, \( h \) is Planck’s constant, and \( V_0 \) is nominal optical frequency.

As may be noticed, the frequency chirp is described by Eq. (3) which shows that the frequency deviation is proportional to the concentration of carriers in the laser active region.

The serious practical drawback of the Eq. (3) is that it relates the chirp to the unobservable carrier concentration, which can not be predicted without the precise knowledge about all the rate equations parameters. Thus it is very useful to relate the chirp to the measurable laser output power. Calculating carrier concentration \( N \) from Eq. (2) and putting it into Eq. (3), the frequency chirp may be related to photon concentration. Ignoring some negligible terms and using Eq. (4) the chirp may be finally related to the laser output power:

\[
\Delta \nu(t) = \frac{\alpha}{4\pi} \left( \frac{1}{P(t)} \frac{dP(t)}{dt} + \kappa P(t) \right),
\]

(5)

where \( \kappa = \Gamma \varepsilon_g / (\eta V_a h V_0) \) is so called adiabatic chirp coefficient. The part of the chirp induced by the time derivate of the power is called the dynamic chirp, and
Are carrier transport effects important for chirp modelling of quantum-well lasers?

that directly proportional to the power is called the adiabatic one.

In case of small signal laser modulation the frequency modulation (FM) efficiency may be determined using Eq. (5). In the frequency domain it takes the form:

$$\frac{\delta \nu(\omega_m)}{\delta I(\omega_m)} = \frac{\alpha}{4\pi} \left( j\omega_m + \kappa \right) \frac{\delta P(\omega_m)}{\delta I(\omega_m)}, \quad (6)$$

where $\delta(\cdot)$ denotes the small signal component of each quantity, $\omega_m$ is angular frequency of laser modulation, $\langle P \rangle$ is mean optical power, and $\delta P(\omega_m)/\delta I(\omega_m)$ is intensity modulation (IM) efficiency.

Thus, having the knowledge about the laser IM behaviour (some kind of model or measured data) only two parameters ($\alpha$ and $\kappa$) are needed to accurate chirp characterization. Some relatively simple measurement methods for determining these parameters are described in many papers [8].

2.2. QW laser modelling

In the QW lasers the carrier concentrations in SCH and QW regions should be distinguished, thus two separate rate equations for the carriers are introduced:

$$\frac{dN_b}{dt} = \frac{I}{eV_w} N_b \tau_{cap} - \frac{N_b}{\tau_{esc}} + \frac{N_w}{\tau_e}, \quad (7)$$

$$\frac{dN_w}{dt} = \frac{N_b}{\tau_{cap}} - \frac{N_w}{\tau_{esc}} - \frac{g_0(N_w-N_T)}{1+\varepsilon_s S} \frac{S}{\tau_e}, \quad (8)$$

where $N_w$ is the carrier concentration in the quantum wells, $N_b$ is some equivalent concentration related with the real SCH carrier concentration $N_s$ by the relation: $N_b = N_s V_s / V_w$, in which $V_s$ and $V_w$ are volumes of SCH and QW respectively. The capturing of the carriers from SCH to QW is characterized by capture time $\tau_{cap}$, and (much less efficient) escaping in the opposite direction by $\tau_{esc}$. The photon density depends only on the $N_w$ concentration, thus:

$$\frac{dS}{dt} = \frac{\Gamma g_0 (N_w - N_T)}{1 + \varepsilon_s S} \frac{S}{\tau_p} + 1 + \frac{\Gamma \beta N_w}{\tau_e}. \quad (9)$$

Differently, the frequency chirp depends both on QW and SCH carrier densities, because the optical field lies in both the regions undergoing carrier concentration variations. Thus the chirp may be expressed as follows [1]:

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\[ \Delta v = \frac{\alpha}{4\pi} \Gamma g_b (N_w - N_{wTH}) + (1 - \Gamma) g_b (N_b - N_{bTH}), \]  

where \( N_{wTH} \) and \( N_{bTH} \) are threshold carrier concentrations in QW and SCH, respectively, \( g_b \) is the coefficient characterizing the efficiency of influence of \( N_b \) on the laser frequency.

Unfortunately, this time the chirp can not be easily related to the intensity modulation, as it was made in Eqs. (5) and (6) for the bulk lasers. Large signal relation, analogous to Eq. (5) is quite complicated, and even after many simplifications needs at least four parameter values to be determined in some way. Similarly, the small signal relation analogous to Eq. (6) is also troublesome and need a large set of parameters \([1]\).

Thus the question of practical importance arise whether the relatively simple model of the laser IM and FM properties, based on the bulk laser rate equations, may be adopted for behavioural (i.e. not strictly connected with physical phenomena) modelling of the QW lasers.

In case of IM characteristics, is was shown in \([7]\) that the effects arising from the carrier accumulation in the SCH may be simply modelled by first order low-pass filter with time constant equal to \( \tau_{cap} \), preceding the bulk model of the inner QW structure. It may be also shown that for QW lasers with the low capture time the difference in the IM properties of models described by Eqs. (1), (2) and (7) … (9) practically vanishes.

\section*{3. SMALL-SIGNAL CONSIDERATIONS}

First the small-signal chirp characteristics arising from the QW laser model based on the rate equations (7) … (10) will be analysed. Using this model and starting from two experimentally verified sets of its parameters, taken from \([9]\), the laser FM efficiency versus modulation frequency was obtained. In some initial investigations it was observed that under the reasonable assumption that \( \tau_{cap} \ll \tau_{exc} \), the \( \tau_{cap} \) becomes the crucial parameter determining both IM and FM differences between the QW and bulk laser model. This is illustrated in Figs. 1 and 2. For the capture time not greater than a few ps the rate equation modelling the SCH region carrier concentration have no considerable influence on laser IM and FM properties, and may be eliminated what leads back to the bulk laser model. For higher values of \( \tau_{cap} \) the IM modulation bandwidth reduces, what is in consistence with above mentioned low-pass-filter-like behaviour of SCH. A bit more complicated is influence of the capture time on FM properties. It may be observed that for low modulation frequencies high values of \( \tau_{cap} \) increase FM
Are carrier transport effects important for chirp modelling of quantum-well lasers?

efficiency, but for high frequencies the influence is opposite. It may be explained as follows: for low modulation frequencies the greater values of $\tau_{\text{cap}}$ lead to higher modulation of carrier concentration in SCH region, which results in higher chirp. For higher frequencies however, the low-pass filtering nature of SCH carrier accumulation reduces the fluctuations of carrier concentrations in SCH and consequently also in QW regions. This way the chirp is strongly reduced (similarly to the IM response).

![Fig. 1. IM efficiency $\frac{\partial P}{\partial f}$ versus modulation frequency and capture time](image1)

![Fig. 2. FM efficiency $\frac{\partial \nu}{\partial f}$ versus modulation frequency and capture time](image2)

![Fig. 3. Comparison of FM efficiency obtained from full QW model and from Eq. (6)](image3)

Now we come back to the practical question whether the Eq. (6) may be used as simple behavioural model of QW laser chirp. In Fig. 3 the FM efficiency obtained from full QW model is compared with that obtained using Eq. (6). Because Eq. (6) is now treated as a behavioural model, the adiabatic chirp coefficient $\kappa$ was trimmed to obtain desired value of the low frequency chirp for each value of the
taken capture time. Also, what should be pointed out, the IM response \( \delta P(\omega_m) / \delta I(\omega_m) \) was modified each time by taking the actual one obtained from full QW rate equations model. As may be noticed a very good agreement between the chirp obtained from full model and from Eq. (6) was obtained, even for frequencies far above the laser relaxation frequency.

Concluding, the QW laser small-signal chirp may be accurately determined by the simple formula given in Eq. (6). However, the accurate IM response (known from any kind of model or measured data) is crucial for good accuracy.

4. LARGE-SIGNAL CONSIDERATIONS

The small-signal FM response is basic laser property in transmission system based on frequency/phase modulation, as some coherent or dispersion-supported systems. But also in case of direct intensity modulation based systems the laser chirp may be important, when it interact with the transmission channel chromatic dispersion. This time however, rather large signal chirp properties should be analysed.

Natural extension of above presented small-signal considerations would be that also large-signal relation between bulk laser FM and IM may be adopted to QW lasers. Following the previous strategy, the large-signal laser chirp was determined by simulating the full QW rate equations model, and next compared with the chirp obtained from Eq. (5). As previously, the adiabatic chirp coefficient \( \kappa \) was trimmed to obtain the best agreement with the full model. The results are illustrated in Fig. 4 for various capture time values. The laser model was driven by the 200 ps long, nearly-rectangular current pulse. One may notice that the chirp obtained from Eq. (5) is extremely close to that resulting from the full model. Only for very large capture time, as 50 ps, some quite small delay (being about 8 ps) may be observed in the chirp obtained from Eq. (5).

Fig. 4. Comparison of time domain chirp evolution obtained from full QW model and from Eq. (6)
A very good agreement of the QW laser chirp characteristics obtained from the full model with that determined from Eqs. (5) and (6) is somewhat surprising, having in the mind that they are derived from the bulk laser model. However, some intuitive explanation may be proposed. First, it should be noticed that using the “bulk” equations (5) and (6) the chirp induced in SCH region is “pushed” into the adiabatic chirp of the active region. This way the changes of the SCH carrier density (which in fact make the SCH chirp component) were in the model “substituted” by the changes of laser optical power, which in case of high-speed modulation would not exactly follow the SCH carrier density. Considering the case of large capture time first, its low-pass filtering feature should be recalled. The 50 ps capture time induces about 3 GHz cut-off which depresses fast changes in SCH carrier density. In this situation the “inner” laser is fast enough, and so the optical power nearly exactly follows the SCH carrier density indeed, what explains the simple model accuracy.

For lower values of capture time the optical power may be more mismatched from SCH carrier density. But on the other hand, small capture time results in small carrier accumulation in the SCH and so small chirp component caused by the SCH region. This way even less accurate modelling of this component has no significant influence on total chirp, thus the simplified model is still quite accurate.

5. EXPERIMENTAL VERIFICATION

Direct measurement of large-signal time-resolved laser chirp is quite complicated and usually suffers from inherent bandwidth limitation introduced by the frequency response of FM/IM converting optical filters. Some indirect, but quite precise verification of chirp modelling may be however performed based on the optical fiber chromatic dispersion. The interaction of the laser chirp with the fiber dispersion causes serious distortions in the time evolution of optical power detected on the fiber end. Comparing the measured signal distortions with that calculated based on the taken chirp model, its adequacy may be verified. The results of such experiment are shown in Fig. 5. The high-speed IM modulated signal (a piece of 10 Gb/s data stream) outgoing the MQW DFB laser (PT3563 type) is illustrated in Fig. 5a. Taking the chirp model in form of Eq. (5), with parameters \( \alpha \) and \( \kappa \) obtained in other measurements, the chirp caused signal distortions after the 20 km long fiber were calculated, and compared with the measurement. As is visible in Fig. 5b, the calculated and measured fiber output signals are practically identical, what proofs the adequate chirp modelling.
Fig. 5. Modulated laser output power (a), and fiber output power corrupted by interplay of the laser chirp and the fiber chromatic dispersion (b)

6. CONCLUSIONS

In the paper the influence of the carrier transport between the SCH and QW regions is analysed in the context of chirp modelling. It was shown that even for high values of carrier capture time, when the transport effects seriously affect the laser IM and FM characteristics, the simple relations coupling intensity modulation with chirp, derived for bulk lasers, may be used. It is of serious practical importance, because it let to determine the chirp from IM characteristics, using the model needing only two parameters: the line enhancement factor $\alpha$ and adiabatic chirp coefficient $\kappa$. Namely, the time domain evolution of chirp may be obtained from measured (or somehow modelled) time domain evolution of the laser output power, by means of Eq. (5). Alternatively, the frequency domain FM transfer function may be obtained from the frequency domain IM transfer function, using Eq. (6). This way in many cases the troublesome full QW laser modelling may be omitted without sacrificing of considerations accuracy.

7. REFERENCES

Are carrier transport effects important for chirp modelling of quantum-well lasers?


