METHODS OF REAL-TIME CALCULATION OF MAXIMUM TIME INTERVAL ERROR

Andrzej Dobrogowski, Michał Kasznia

Abstract – In this paper the methods enabling assessment of Maximum Time Interval Error (MTIE) in the real time are presented and compared. The idea of real-time quasi-parallel computation of MTIE is described. Then some different methods enabling real-time MTIE calculation are presented. The results of experimental tests performed for different data sequences are presented and discussed.

Index Terms – Timing signal, time error, maximum time interval error.

I. INTRODUCTION

Maximum Time Interval Error (MTIE) is commonly used for evaluation of the quality of telecommunication network timing signal [1, 2, 3]. The MTIE is usually computed for a series of observation intervals, starting from some \( \tau_{\text{min}} \) until some \( \tau_{\text{max}} \). The computation of the MTIE point estimate usually follows the measurement of a sequence of time error samples, performed at some network interface. Long-term time error measurement and long time of the plain computation of the parameter cause that the whole evaluation process is very time-consuming. In order to make the computation shorter, several time effective methods, enabling rather fast MTIE calculation, were proposed and described. Some methods were proposed by the authors of this paper [5, 6, 7, 8].

Some features of the methods (sequential data reduction, long window’s shifts) enabled to formulate the idea of on-line MTIE computation, which is performed in the real time, during the measurement of time error samples, and parallel for several observation intervals [6]. In order to calculate the MTIE estimate simultaneously for several observation intervals in the real time, all necessary operations should be performed in the time period between two sampling instants, i.e. during the sampling interval \( \tau_0 \).

The ability of real time assessment depends on the following conditions: number and length of the observation intervals considered, computational power of the measurement equipment, and time error data behavior. In the paper the results of computation tests [9, 10] of the methods described are presented and discussed.

II. MAXIMUM TIME INTERVAL ERROR

The point estimate of the Maximum Time Interval Error is defined in the international standards as the maximum peak-to-peak time error variation of a given timing signal, with respect to an ideal timing signal within a particular time period [1, 2, 3]. If the results of time error function measurements \( x(t) \) take the form of \( N \) equally spaced samples \( \{x_i\} \), MTIE can be estimated from the formula

\[
\text{MTIE}(\tau_n) = \max \left( \max_{1 \leq k \leq N-n, k \leq i < k+n} x_i - \min_{k \leq i < k+n} x_i \right)
\]

where \( \{x_i\} \) is a sequence of \( N \) samples of time error function \( x(t) \) taken with sampling interval \( \tau_0 \), \( \tau = n \tau_0 \) is an observation interval, and \( n \) can change from 1 till \( N-1 \) depending on the considered values of observation intervals.

Following directly the formula (1) in order to find the estimate of MTIE for the observation interval \( \tau \), all intervals having the width of \( \tau \), existing in the sequence of \( N \) time error samples, must be reviewed. The window having the width of \( \tau = n \tau_0 \) and including \( n+1 \) samples is set at the beginning of the data sequence \( \{x_i\} \) and then it is shifted with the step of \( \tau_0 \) to the end of the sequence. For each window’s location the peak-to-peak value of time error in the window is found. The maximum peak-to-peak value found for all existing locations of the window is the value of \( \text{MTIE}(\tau) \) estimate. The process of window reviewing does not depend on the data value. The complexity of calculation grows with \( n \) and therefore the direct method is really time-consuming. The idea of direct search (plain computation) of MTIE is presented in Fig. 1.

![Figure 1. The idea of direct search for MTIE](image-url)
reduce the complexity of computation. In the process of the MTIE search using the extreme fix (EF) method some window’s locations are excluded from inspection if the peak-to-peak value for each of these locations is not greater than the value found until now, or if a greater peak-to-peak value may be found for the successive window’s locations [5]. The EF method is based on fixing the positions of minimum and maximum samples for a given window’s location. The general rule is that the next window’s location is originated at the first extreme previously found. The idea of EF method is presented in Fig. 2.

The methods with sequential data reduction were established for the MTIE assessment performed for a series of observation intervals, starting from some τ_{\min} until some τ_{\max}. The methods follow the suggestion according to which during the MTIE search process for some observation interval τ_i we find the extreme samples for some window’s location from the set of extreme samples found previously during the MTIE search for the smaller observation interval τ_{i+1} (τ_{i+1} > τ_i). Therefore, we can reduce the number of time error samples used for the MTIE calculation. Two methods with sequential data reduction were proposed by the authors of this paper. The first method, called EFSDR, uses extreme fix search for the raw data sequence as well as for the reduced data [7]. Unfortunately, it may produce errors and the final MTIE value may be underestimated. In order to avoid the errors, the rules of the extreme searching were changed. The second method – direct search with sequential data reduction (DSDR) – uses plain computation at each level of the procedure (for raw and reduced data) [8]. Another method was proposed by Bregni and Maccabruni in [4]. This method uses binary decomposition of the data sequence. At the first step of the calculation process the 2-samples windows are considered. Two neighboring samples are compared and the maximum and minimum values for each pair are selected. The maximum of the difference between them within one window is the MTIE for the 2-samples observation interval. At the second step previously selected extreme values are used for finding the maximum and minimum for 4-samples windows (the comparison of the extremes for the neighboring pairs is performed only). Then the MTIE is searched. At the successive steps the next windows (containing increasing number of samples) are considered by the creation from the previously analyzed windows, and the data reduction process proceeds. The method with binary decomposition is characterized by the limitation: the lengths of observation intervals considered should be a power of 2.

III. REAL-TIME COMPUTATION

The formula of the MTIE estimator allows to perform the calculation of the parameter in the real-time, during the time error measurement. Therefore we can observe the value of the parameter during the long lasting measurement process. Any possible wrong behavior of the analyzed signal (recognized, if MTIE exceeds the limit) enables applying proper activity of a maintenance team.

A general procedure of the real-time quasi-parallel MTIE calculation for a series of observation intervals is as follows [6]:
1. Measure a new time error sample.
2. Compare the new sample with current maximum and minimum.
3. If current window’s location is filled out with samples, fix the extremes for this location.
4. Check if the current window’s location is filled out with samples for the next longer observation interval.
5. If so, find the extremes for this location and check the conditions for the next longer observation interval.
6. When the measurement is finished, continue the computation for the remaining locations of each longer observation interval.

The choice of the algorithm suited for the real-time parallel calculation is very important. Because all necessary operations have to be performed in the time period between two successive sampling instants (during the sampling interval τ_0), the calculation algorithm should be time effective in order not to exceed the sampling interval. Authors of the paper have adopted three time effective methods for real-time calculations [9, 10]. The principles of these methods will be presented below.

A. Real-time direct search with sequential data reduction

The real-time computation using DSDR method for the first (shortest) observation interval τ_{\min} begins with the first measured time error sample. Each new sample measured is compared with current maximum and minimum values, until the first window’s location is filled out by the samples. Then the extreme values for this location are fixed. Each successive measured sample creates a new window’s location. The extreme samples found for each window’s location create new data sequences with reduced number of items. The items of reduced data sequences are used for the MTIE estimate calculation for the observation intervals longer than τ_{\min}. The first location of the next longer window is not analyzed until all samples situated in this location are reviewed by the preceding window. The example of computation using DSDR method is presented in Fig. 3 [9].

B. Real-time extreme fix method

In the case of real-time calculation using EF method, the computation procedures for each observation interval run independently. Window’s locations of longer observations intervals are analyzed after filling out by the samples without waiting for the analysis by the preceding shorter windows. All windows are activated after filling out their first locations by the samples. The extremes found for some
observation interval do not affect the calculation process for other observation intervals. The example of real-time MTIE calculation – early stage of the process – for observation intervals having 6, 8, and 10 samples using EF method is presented in Fig. 4. Ten samples were measured and all windows are activated. The extreme samples (black and white stars) in the relevant window’s locations are found and the next window’s locations are set (dashed line). At the end of measurement, after analysis of the last window’s locations, the parameter’s value for each observation interval is known without any delay [9].

The adoption of binary decomposition method for computation in the real time was performed in [10]. The computation starts with the first sample measured. Each successive measured sample is compared with the sample measured previously. The result of the comparison – pair of maximum and minimum – is stored for further analysis. The difference between these values (peak-to-peak value) is compared with current maximum and minimum – is stored for further analysis.

The difference between these values (peak-to-peak value) is compared with current maximum and minimum – is stored for further analysis. The difference between these values (peak-to-peak value) is compared with current maximum and minimum – is stored for further analysis.

As result, the pair of 5 and 1 is stored for further operations. The peak-to-peak values are computed and compared for appropriate observation intervals: 2, 4, and 8-samples windows.

C. Real-time computation with binary decomposition

The results of the experimental tests of the real-time MTIE computation using EF and DSDR methods were presented in [9]. The calculations were performed off-line but the on-line work was imitated. Three different time error sequences were used: first represents one of typical noises of the timing signals – white phase modulation (WPM); second was the result of comparison of two different GPS disciplined oscillators; third was the result of comparison of two independent internal oscillators of some measurement system (MSG). The time error samples were taken with the sampling interval \( \tau_0 = 1/30 \) s. The MTIE values were computed using both methods for the series of observation intervals arranged in the logarithmic scale between 0.1 s and 1000 s, starting from 0.1 s. The observed quantity was the maximum time spent for calculation for one sampling interval. The calculations were performed using personal computer with Pentium IV 3.0 GHz microprocessor. The results of calculations are presented in Table 1 and Table 2. The results have showed very good behavior of the EF method for small number of observation intervals (up to 10) from the range 0.1-10 s. For longer observation intervals analyzed simultaneously and their greater ranges, the results using DSDR method were much better. Reduction of intervals’ number (lower rows in Table 1 and 2) within the same range does not improve the results for DSDR – the observed time is longer than previously for MSG and GPS sequences. In such a case, the data reduction process is not as effective as in the case of greater number of intervals. As result, the longest observation intervals work on more numerous sequences. Similar reduction of intervals’ number in the case of EF method does not reduce the time significantly. The main reason of long computation time is the necessity of whole window’s location’s review for the longest observation interval.

IV. RESULTS OF EXPERIMENTAL TESTS

The results of the experimental tests of the real-time MTIE computation using EF and DSDR methods were presented in [9]. The calculations were performed off-line but the on-line work was imitated. Three different time error sequences were used: first represents one of typical noises of the timing signals – white phase modulation (WPM); second was the result of comparison of two different GPS disciplined oscillators; third was the result of comparison of two independent internal oscillators of some measurement system (MSG). The time error samples were taken with the sampling interval \( \tau_0 = 1/30 \) s. The MTIE values were computed using both methods for the series of observation intervals arranged in the logarithmic scale between 0.1 s and 1000 s, starting from 0.1 s. The observed quantity was the maximum time spent for calculation for one sampling interval. The calculations were performed using personal computer with Pentium IV 3.0 GHz microprocessor. The results of calculations are presented in Table 1 and Table 2. The results have showed very good behavior of the EF method for small number of observation intervals (up to 10) from the range 0.1-10 s. For longer observation intervals analyzed simultaneously and their greater ranges, the results using DSDR method were much better. Reduction of intervals’ number (lower rows in Table 1 and 2) within the same range does not improve the results for DSDR – the observed time is longer than previously for MSG and GPS sequences. In such a case, the data reduction process is not as effective as in the case of greater number of intervals. As result, the longest observation intervals work on more numerous sequences. Similar reduction of intervals’ number in the case of EF method does not reduce the time significantly. The main reason of long computation time is the necessity of whole window’s location’s review for the longest observation interval.
Data reduction using binary decomposition results in better computation of MTIE. The regularly running sequential method, especially for the MSG time error sequence and for sequences. The operations performed using binary exceed the sampling interval in the case of each time error calculation using both methods is presented in Table 3 [10].

The maximum operation interval took the values of 32 samples (1092.2 s). Therefore the computations were performed for 2-samples interval is not necessary in this case). The maximum observation interval took the values of 3 samples (17.03 s), 4096 samples (1136.5 s), and 32768 samples (3.023 s). Therefore the computations were performed for 2, 8, 11, and 14 simultaneously analyzed observation intervals using DSDR method, and for 5, 9, 12, and 15 intervals using binary decomposition. The time of using both methods is presented in Table 3 [10].

Table 1. Time of calculation using DSDR method

<table>
<thead>
<tr>
<th>Number of intervals</th>
<th>Range of intervals [s]</th>
<th>WPM</th>
<th>GPS</th>
<th>MSG</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.1-1</td>
<td>0.0028</td>
<td>0.0022</td>
<td>0.0027</td>
</tr>
<tr>
<td>11</td>
<td>0.1-10</td>
<td>0.0044</td>
<td>0.0044</td>
<td>0.0071</td>
</tr>
<tr>
<td>16</td>
<td>0.1-100</td>
<td>0.0055</td>
<td>0.0227</td>
<td>0.0247</td>
</tr>
<tr>
<td>21</td>
<td>0.1-1000</td>
<td>0.0071</td>
<td>0.0110</td>
<td>0.0549</td>
</tr>
<tr>
<td>3</td>
<td>0.1-1</td>
<td>0.0017</td>
<td>0.0017</td>
<td>0.0017</td>
</tr>
<tr>
<td>5</td>
<td>0.1-10</td>
<td>0.0022</td>
<td>0.0033</td>
<td>0.0055</td>
</tr>
<tr>
<td>7</td>
<td>0.1-100</td>
<td>0.0028</td>
<td>0.0055</td>
<td>0.0330</td>
</tr>
<tr>
<td>9</td>
<td>0.1-1000</td>
<td>0.0033</td>
<td>0.0220</td>
<td>0.0934</td>
</tr>
</tbody>
</table>

Table 2. Time of calculation using EF method

<table>
<thead>
<tr>
<th>Number of intervals</th>
<th>Range of intervals [s]</th>
<th>WPM</th>
<th>GPS</th>
<th>MSG</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.1-1</td>
<td>0.0006</td>
<td>0.0008</td>
<td>0.0007</td>
</tr>
<tr>
<td>11</td>
<td>0.1-10</td>
<td>0.0027</td>
<td>0.0034</td>
<td>0.0039</td>
</tr>
<tr>
<td>16</td>
<td>0.1-100</td>
<td>0.0165</td>
<td>0.0209</td>
<td>0.0364</td>
</tr>
<tr>
<td>21</td>
<td>0.1-1000</td>
<td>0.1412</td>
<td>0.1455</td>
<td>0.1802</td>
</tr>
<tr>
<td>3</td>
<td>0.1-1</td>
<td>0.0006</td>
<td>0.0005</td>
<td>0.0007</td>
</tr>
<tr>
<td>5</td>
<td>0.1-10</td>
<td>0.0021</td>
<td>0.0026</td>
<td>0.0036</td>
</tr>
<tr>
<td>7</td>
<td>0.1-100</td>
<td>0.0134</td>
<td>0.0154</td>
<td>0.0318</td>
</tr>
<tr>
<td>9</td>
<td>0.1-1000</td>
<td>0.1324</td>
<td>0.1329</td>
<td>0.1346</td>
</tr>
</tbody>
</table>

Similar experimental tests for the same conditions were performed for binary decomposition method comparing with DSDR in [10]. Because we cannot compute the MTIE using binary decomposition for arbitrary chosen observation intervals, the calculations were performed for the intervals having the power of 2 samples. The starting (smallest) observation interval had 2 samples for binary decomposition and 4 samples for DSDR method (the computation for 2-samples interval is not necessary in this case). The maximum observation interval took the values of 32 samples (which corresponds with 1.03 seconds), 512 samples (17.03 s), 4096 samples (1136.5 s), and 32768 samples (3.023 s). Therefore the computations were performed for 2, 8, 11, and 14 simultaneously analyzed observation intervals using DSDR method, and for 5, 9, 12, and 15 intervals using binary decomposition. The time of calculation using both methods is presented in Table 3 [10].

Table 3. Time of calculation (in sec) using DSDR and binary decomposition (BIN) methods

<table>
<thead>
<tr>
<th>Number of intervals</th>
<th>WPM</th>
<th>GPS</th>
<th>MSG</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>DSDR BIN</td>
<td>DSDR BIN</td>
<td>DSDR BIN</td>
</tr>
<tr>
<td>45</td>
<td>0.0014</td>
<td>0.0011</td>
<td>0.0016</td>
</tr>
<tr>
<td>89</td>
<td>0.0027</td>
<td>0.0016</td>
<td>0.0041</td>
</tr>
<tr>
<td>1(12)</td>
<td>0.0038</td>
<td>0.0019</td>
<td>0.0061</td>
</tr>
<tr>
<td>14(15)</td>
<td>0.0044</td>
<td>0.0024</td>
<td>0.0134</td>
</tr>
</tbody>
</table>

The maximum time of operations performed for one sampling instant using binary decomposition does not exceed the sampling interval in the case of each time error sequences. The operations performed using binary decomposition was less time consuming than using DSDR method, especially for the MSG time error sequence and for greater ranges of observation intervals.

V. CONCLUSIONS

All methods described enable real-time quasi-parallel computation of MTIE. The regularly running sequential data reduction using binary decomposition results in better behavior in comparison with two other methods, especially for the widest range of observation intervals. However the required dimensions of observation intervals limit the application of the binary decomposition method. The suspension of computation for longer observation interval using EF or DSDR methods could be considered in order to reduce the time of operations performed for one sampling interval. Such suspension could be realized, when specified number of processes of whole window’s location search performed at once is exceed. The analysis of suspended intervals may be performed later, when less operations is made, or at least, when the measurement is finished.

This work was supported by the Ministry of Science and Higher Education in the frame of the project number 1645/B/T02/2007/33.

REFERENCES


AUTHORS’ NOTE

Andrzej Dobrogowski – Chair of Telecommunications Systems and Optoelectronics, Poznań University of Technology, ul. Polanka 3, 60-965 Poznań, Poland, dobrog@et.put.poznan.pl, area of interest: synchronization in telecommunication networks and systems, optical networks, estimation of signals’ parameters.

Michał Kasznia – Chair of Telecommunications Systems and Optoelectronics, Poznań University of Technology, ul. Polanka 3, 60-965 Poznań, Poland, mkasznia@et.put.poznan.pl, area of interest: synchronization in telecommunication networks and systems, analysis of the quality of timing signals.