Performance Analysis of LDPC Coded OFDM System

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Abstract—The standard of 802.11n adopts Low Density Parity Check Codes (LDPCC) as an optional coding scheme. This coding offers high reliability, which is important factor for modern systems. To illustrate the performance of LDPC coded OFDM system over AWGN or Channel with Rayleigh fading, a simulation model was implemented. This paper presents impact of modulation and coding schemes on the LDPC codes efficiency. Also average number of needed iterations to successfully decode LDPC codeword was measured.

Index Terms—LDPCC, OFDM, IEEE 802.11n

I. INTRODUCTION

The goal of wireless systems e.g. 4G, IEEE 802.11n/802.16 standards is to reliably send data with a low frame error rate (FER). To achieve that, several methods are used. In this paper Low Density Parity Check Codes are considered. It appears promising in fulfilling the growing demand for future high data rate systems.

Low density parity check codes were discovered by Gallager[1] and rediscovered by MacKay[2]. Thanks to the low decoding complexity and performance, which is close to the Shannon capacity limit [3], they are very attractive to further research.

The simulation model, based on the IT++ library [5], was implemented to simulate IEEE802.11n [8] WLAN system with LDPC coding. The paper is organized as follows. First the simulation model is described. Next, the simulation of the different code length and different modulation is presented. After that, the performance of the system is compared with the performance of a system with changed constellation mapping. The final section presents conclusions.

II. SIMULATED SYSTEM

At the transmitter side in simulated model (depicted in Fig. 1) LDPC codeword is modulated QAM. The mapper chooses the constellation point for the transmitted symbol given. After that the OFDM transmission is performed over channel with Rayleigh fading. At the receiver side received signal is transformed into frequency domain and Log-MAP demodulator computes log likelihood ratio metrics (LLR) for the received bits. It is assumed ideal channel state information at the receiver side. These LLR metrics are the input to the LDPC decoder. In this paper Belief Propagation (BP) [1] decoding algorithm is considered.

![Simulated System Model](image)

Fig. 1. The simulated system's model

A. Low Density Parity Check Codes

LDPC code is a linear block code specified by a very sparse parity check matrix. Irregular LDPC codes are a generalization of LDPC codes proposed by Gallager. In comparison to regular codes, for the irregular codes the degrees of variable and check nodes do not need to be constant. For the irregular codes degree distribution functions are often used for description purpose [3][13]. From the perspective of the edge connected to variable nodes the function is defined as follows:

\[
\lambda(x) = \sum_{i=1}^{d_v} \lambda_i x^{i-1},
\]

where \(\lambda_i\) is a fraction of edges connected to variable nodes of degree \(i\). From the perspective of edges connected to check nodes:

\[
\rho(x) = \sum_{i=2}^{d_c} \rho_i x^{i-1},
\]

where \(\rho_i\) is a fraction of edges connected to check nodes of degree \(i\). Having degree distribution function defined, the rate of the code is equal to:

\[
R(\lambda, \rho) = 1 - \frac{\sum_{i=1}^{d_v} \lambda_i}{\sum_{i=2}^{d_c} \rho_i} = 1 - \frac{1}{\int_0^1 \rho(x) dx} \int_0^1 \lambda(x) dx.
\]

LDPC codes could be represented by a bipartite graph, called Tanner graph, in which N variable nodes corresponds to the set of parity check constraints and each edge corresponds to a non-zero entry in the parity check matrix. LDPC codes can achieve very good performance when they are decoded with the belief-propagation (BP) or the sum–product algorithm (SPA) [4]. The algorithms iteratively update the a posteriori probabilities of each bit in the codeword.
LDPC codes are introduced as an optional coding mode in IEEE802.11n [10]. The IEEE 802.11n LDPC codes are based on block-structured LDPC codes with circular block matrices [6], i.e., the entire parity check matrix can be partitioned into an array of block matrices. Each block matrix is either a zero matrix or a right cyclic shift of an identity matrix. Since the parity check matrices are extremely large and thus difficult to display, certain notational shortcuts are taken to represent these matrices.

B. Orthogonal Frequency Division Multiplexing

Orthogonal Frequency Division Multiplexing (OFDM) is widely used in communication systems as WLAN, DVB, etc. It is well suited to wideband systems in frequency selective fading environments, because only a few subcarriers are impacted by a deep fade or narrow band interference, which can be protected by forward error correction. In addition, OFDM is bandwidth efficient, since a nearly square power spectrum can be created with narrow subcarriers with each subcarrier supporting a constellation with many bits per symbol. Because these subcarriers are orthogonal, they do not interfere with one another [12].

OFDM signals are generated using the fast Fourier transform (FFT). FFT allows individual channels to maintain their orthogonality (distance, to adjacent channels). Moreover using FFT, data symbols can be reliably extracted and multiple subchannels can overlap in the frequency domain, which increases spectral efficiency. OFDM technique is a very robust against multipath fading.

According to IEEE 802.11n the size of FFT is equal to 64.
This standard offers two types of bandwidth: 20MHz (legacy) and 40MHz. In comparison to IEEE 802.11a two data subcarriers are added to each end (if it is used 20MHz spectrum). The total number of used subcarriers is 56. The data is carried by subcarriers −28 to −22, −20 to −8, −7 to −1, 1 to 6, 8 to 20, and 22 to 28. The pilot subcarriers are located in the same subcarriers as legacy, −21, −7, 7, 21. Also as with legacy, there is a null at DC [8][11].

![OFDM signal for 20MHz bandwidth](image)

**Fig. 2. IEEE802.11n OFDM signal for 20MHz bandwidth.**

### III. SIMULATION

A. Assumptions

The simulation model was written in C++ using IT++ library version 4.0.7. Ideal synchronization at the receiver side was assumed. The maximum number of BP algorithm iterations is 50. Moreover, after every iteration, syndrome [14] is computed to check if the currently decoding codeword is valid.

A set of LDPC codes used for this simulation are described in [8]. This means, that transmitting is organized as follows. The encoded LDPC codeword is modulated to complex symbols using 4-QAM, 16-QAM or 64-QAM. Next pilots and zero padding signals are inserted to the input symbols. After that IFFT is performed to convert to time domain and construct OFDM symbols (described in more details in previous section). Finally a guard interval (16 subcarriers) is inserted before each OFDM symbol.

Prepared in this matter frame is transmitted through simulated channel with Rayleigh fading or AWGN. The fading channel is based on the 9-tap exponential model with the average power of the path described in [9] as a model B. As a simplification, there was no antenna correlation assumed. The normalized Doppler frequency was set to 0.01 Hz.

At the receiver side, the incoming signal is transformed back to the frequency domain by FFT, after guard interval removal. These symbols are demapped by Log-MAP algorithm implemented in IT++ library. Log Likelihood Ratio (LLR) of the received signal is calculated and fed into LDPC decoder, where BP algorithm is performed.

B. Simulation Results

Various types of modulation and code rate were simulated. The measured performance of LDPC codes in AWGN channel is presented in Fig. 3 (Bit Error Rate - BER) and in Fig. 4 (Frame Error Rate - FER).

The graphs showing that, for the same code rate BER (Fig. 3) and FER (Fig. 4) approach the same level for lower SNR, as data length increase. For example code with rate R=1/2, modulation type 16QAM code, n=1296 requires approximately 0.3 dB lower SNR to achieve the same BER or FER than the same code, but the length equal to n=648. This is caused by averaging bit error rate with increasing data bits number.

However gain from the codeword length is paid by higher number of needed iterations. It is also presented that average number of successful iterations is decreasing faster for longer codeword with raising SNR. Fig. 5 presents this fact. If the codeword was successfully decoded, the number of iterations was stored. Sum of all stored iterations divided by number of all successfully decoded frames are presented on the Fig. 5 and Fig. 8.

On the Fig. 4 simulation results from [15] are marked with a dashed line. The achieved results are slightly different, because in [15] were used OFDM from standard IEEE 802.11a. Additional the frame length is 200 bytes, which
obviously has an impact on the performance. For presented simulation frame length is equal to codeword length (n).

During simulations in Rayleigh fading channel the same phenomenon as for simulation in AWGN channel were observed. BER (Fig. 6) and FER (Fig. 7) approach the same level for lower SNR values if the codeword is longer. For example if the code rate is R=3/4, modulation 16-QAM, codeword with length n=1296 requires approximately 0.9dB lower SNR than codeword with length n=648 to achieve PER=10^{-2}.

However the curves are decreasing slower with SNR increasing in Rayleigh fading channel than in AWNG channel. Moreover the average number of successful iterations is also decreasing slowly with increasing SNR values (Fig. 8).

IV. CONCLUSIONS

All achieved results shows that frame size does affect BER on LDPC codes performance, which make it a parameter to control BER for the system using LDPC codes, as well as code rate and modulation type. It was also observed, that longer codeword decoding demands higher number of BP algorithm iterations, which is also an important argument, whereas choosing LDPC for the transmission in the channel given. It is also worth to consider amount of iterations when choosing LDPC codeword length, since it makes significant difference for low SNR values.
V. REFERENCES


Robert Orzechowski received M.Sc. degree in electronics and telecommunications from the Poznan University of Technology, Poznan, Poland in 2006. Since 2008, he has been working toward the Ph.D. degree at the Department of Electronics and Telecommunications, Poznan University of Technology, Poland. Currently he holds a position as a Senior Software Engineer at Motorola GSG Poland, Krakow, where he works with Public Safety telecommunications systems. His research interests include error-control coding and wireless communications.